Separation of the Magnetic Field at Mercury

Modeling Mercury's Interaction

Planetary Magnetic Field

\[
\vec{B}_{\text{planet}} = \nabla \left( \frac{\vec{m} \cdot \vec{r}}{4\pi r^3} + \frac{r^T \cdot Q \cdot \vec{r}}{8\pi r^5} + \ldots \right)
\]

\[\vec{m}_{\text{dipole}} \quad Q_{\text{quadrupole}}\]

\[\sigma(r)_{\text{conductivity}}\]

Simulation

XY-plane total velocity at t=15500

MHD-Equations

\[
\begin{align*}
\partial_t \rho &= -\nabla \cdot \rho \vec{u} + D \nabla^2 \rho \\
\partial_t \vec{u} &= - (\vec{u} \cdot \nabla) \vec{u} - \frac{1}{\rho} \nabla p - \frac{1}{\rho} \vec{j} \times \vec{B} - \vec{g} = 0 \\
\partial_t p &= - (\vec{u} \cdot \nabla) p - \gamma p \nabla \cdot \vec{u} + D_p \nabla^2 p \\
\partial_t \vec{B} &= - \nabla \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B} \\
\vec{j} &= \nabla \times \vec{B}
\end{align*}
\]

Solar Wind

\[\rho_{SW}, p_{SW}, \vec{u}_{SW}, \vec{B}_{SW}\]
Time-Dependent Reconstruction

- Magnetosphere of Mercury varies due to solar wind conditions
- Induction at Mercury can have significant impact to the magnetic field
- Non-linear processes such as reconnection
  → Non-linear effects are important
Time-Dependent Reconstruction

\[ \vec{m}_{\text{dipole}} \quad Q_{\text{quadrupole}} \]

\[ \sigma(r)_{\text{conductivity}} \]

Simulation

Cost-Function G

\[ \vec{u}_{SW}(t) \quad \vec{B}_{SW}(t) \]

\[ \rho_{SW}(t) \quad p_{SW}(t) \]

Time-dependent solar wind

Time-dependent data

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Using Solar Wind Statistics

Solar wind distribution at Earth's orbit

- $P(\rho_{SW})$
- $P(u_{SW})$
- $P(B_{z,SW})$
- $P(B_{SW})$
Time-Dependent Reconstruction

\[ \vec{m}_{\text{dipole}} \quad Q_{\text{quadrupole}} \quad \sigma(r)_{\text{conductivity}} \]

\[ \vec{u}_{SW}(t) \quad \vec{B}_{SW}(t) \quad \rho_{SW}(t) \quad p_{SW}(t) \]

Cost-Function

\[ P(\vec{u}_{SW}) \quad P(\vec{B}_{SW}) \quad P(\rho_{SW}) \quad P(p_{SW}) \]
Time-dependent reconstruction

Advantage: Planetary parameters as well as the time-dependent solar wind can be reconstructed

Problem: Many simulation runs need to be performed  
→ time-consuming

How can we fasten the reconstruction?

a) Fasten the minimization  
→ Adjoint approach

b) Speed up the MHD simulation  
→ TSE-MHD
a) Adjoint MHD model

1. Numerical Code

\[ \vec{m}_{dipole} \]
\[ \vec{u} = F \left( \vec{u}_P \left( \vec{m} \right) \right) \]
\[ (u_{n+1,i} = u_{n,i} \cdot \frac{\delta t}{\delta x} \left( \frac{u_{n,i} \cdot u_{n,i+1}}{u_{n,i}} \right) + ...) \]
\[ G \left( \vec{u} \right) \]

Set values for parameters
Calculate flow solution
Calculate cost-function

2. Derivative of cost function

\[ \frac{dG \left( \vec{u} \right)}{d \vec{m}} = \frac{\partial G \left( \vec{u} \right)}{\partial \vec{u}} \cdot \frac{\partial \vec{u}}{\partial \vec{u}_P} \cdot \frac{\partial \vec{u}_P}{\partial \vec{m}} \]

normal
adjoint
b) TSE MHD simulation

Full MHD Code

Series Expansion MHD Code

$10^7$ times faster (Nabert et al. 2013)

(Next step: Projection)
Earth's Dipole Reconstruction

Reconst. from bow shock positions (using THEMIS solar wind observations)
Reconst. from magnetosheath data (OMNI solar wind data)
Reconst. from magnetosheath data (average SW conditions)
Solar wind reconstruction

![Graph showing solar wind density, velocity, and magnetic field over time]
Conclusion

- It is important to consider the time-dependence of the system.
- An **adjoint approach** is an efficient method to calculate the gradient of the minimization.
- Fast **MHD simulations** can be done by the TSE-MHD code.

$$\frac{dG(\mathbf{u})}{d\mathbf{m}} = \frac{\partial G(\mathbf{u})}{\partial \mathbf{u}} \cdot \frac{\partial \mathbf{u}}{\partial \mathbf{u}_p} \cdot \frac{\partial \mathbf{u}_p}{\partial \mathbf{m}}$$
Some recent analysis

